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Updated NLL results for $\bar{B} \to X_{s,d}\gamma$ in and beyond the SM*

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Abstract. We present general model-independent formulae for the branching ratios and the direct tagged CP asymmetries for the inclusive $\bar{B} \to X_d\,\gamma$ and $\bar{B} \to X_s\,\gamma$ modes. We also update the corresponding SM predictions.

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1 Introduction

In the near future more precise data on the inclusive decay $B \to X_s\gamma$ is expected from the B factories, but also the present experimental accuracy already reached the 10% level as reflected [1,2,3,4,5,6] in the world average of the present measurements:

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.34 \pm 0.38) \times 10^{-4}. \tag{1}$$

In addition, direct CP asymmetries within this mode are now within experimental reach [7,8]:

$$A_{\mathrm{CP}}(\bar{B} \to X_s\gamma) = \begin{cases} -0.079 \pm 0.108_{\mathrm{stat}} \pm 0.022_{\mathrm{syst}} \\ -0.004 \pm 0.051_{\mathrm{stat}} \pm 0.038_{\mathrm{syst}} \end{cases} \tag{2}$$

In the first measurement of CLEO there is a small contamination of the $\bar{B} \to X_d\gamma$ mode.

All these measurements are compatible with the standard model (SM) predictions and thus lead to severe constraints on new physics models [9,10,11,12,13,14], which represents very valuable information for the direct search for physics beyond the SM (for recent reviews, see [15,16, 17]).

A direct measurement of the inclusive $\bar{B} \to X_d\gamma$ mode is rather difficult, but perhaps still within the reach of the present high-luminosity B factories. However, the CP violation within that mode can be perhaps tested indirectly by an untagged CP measurement (see below).

In this letter we present general model-independent formulae for the branching ratios and the direct tagged CP asymmetries for the inclusive $\bar{B} \to X_{s,d}\gamma$ modes as explicit numerical expressions for these observables as functions of Wilson coefficients and CKM angles. The extraction of the latter from experimental data depends critically on the assumptions about the presence and the structure of new physics contributions to several key observables.

For this purpose we update and generalize the SM results at NLL level given in [18,19] and [20,21,22] in order to accommodate new physics models with new CP-violating phases and also implement several improvements. For a detailed discussion of our results we refer the reader to a forthcoming paper [23].

2 NLL predictions

The general effective hamiltonian that governs the inclusive $\bar{B} \to X_q\gamma$ decays ($q = d, s$) in the SM is

$$H_{\mathrm{eff}}(b \to q\gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{tq}^* \times \times \left(\sum_{i=1}^{8} C_i\mathcal{O}_i + \epsilon_q \sum_{i=1}^{2} C_i(\mathcal{O}_i - \mathcal{O}_i^u) \right) \tag{3}$$

where $\epsilon_q = (V_{ub}V_{uq}^*)/(V_{tb}V_{tq}^*)$ and the most relevant operators are:

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{q}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L),\\
\mathcal{O}_1^u &= (\bar{q}_L\gamma_\mu T^a u_L)(\bar{u}_L\gamma^\mu T^a b_L),\\
\mathcal{O}_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)\,,\\
\mathcal{O}_2^u &= (\bar{s}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu b_L)\,,\\
\mathcal{O}_7 &= (e/16\pi^2)m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu}\\
\mathcal{O}_8 &= (g_s/16\pi^2)m_b(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G^a_{\mu\nu}\,.
\end{aligned}$$

The subscripts L and R refer to left- and right-handed components of the fermion fields. In $b \to s$ transitions the contributions proportional to ϵ_s are rather small, while in

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$b \to d$ decays the ϵ_d term is of the same order as the first term in effective hamiltonian.

Regarding the input parameters we focus here on the issue of the charm mass definition in the matrix element of $\mathcal{O}_2$: In [18], it is argued that all the factors of m_c come from propagators corresponding to charm quarks that are off-shell by an amount $\mu^2 \sim m_b^2$. It seems, therefore, more reasonable to use the $\overline{\text{MS}}$ running charm mass at a scale μ in the range (m_c, m_b). The reference values of the charm and bottom masses are $m_c = m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}}) = (1.25 \pm 0.10)\,\text{GeV}$ and $m_b = m_b^{1S}$, where the $1S$ mass of the b quark is defined as half of the perturbative contribution to the Υ mass as usual: $m_b^{1S} = (4.69 \pm 0.03)\,\text{GeV}$. We first fix the central value of $m_c = 1.25\,\text{GeV}$ and vary μ; then we add in quadrature the error on m_c ($\delta_{m_c} = 8\%$). The resulting determination is:

$$\frac{m_c}{m_b} = 0.23 \pm 0.05\,. \tag{4}$$

The pole mass choice corresponds, on the other hand, to $\frac{m_c}{m_b} = 0.29 \pm 0.02$. Note that the question whether to use the running or the pole mass is, strictly speaking, a NNLL issue. The most conservative position consists in accepting any value of m_c/m_b that is compatible with any of these two determinations: $0.18 \le m_c/m_b \le 0.31$. Taking into account our experience on higher-loop computations, we are led to the educated guess that the central value $m_c/m_b = 0.23$ represents the best possible choice, but we allow for a large asymmetric error that fully covers the above range (and that reminds us of this problem that can be solved only via a NNLL computation):

$$\frac{m_c}{m_b} = 0.23^{+0.08}_{-0.05}\,. \tag{5}$$

We present our SM updates for two different energy cuts within the photon spectrum $E_0 = (1.6\,\text{GeV}, m_b/20)$. There are four sources of uncertainties: the charm mass (δ_{m_c/m_b}), the CKM factors ($\delta_{\text{CKM}}(s) = 0.5\%$, $\delta_{\text{CKM}}(d) = 11\%$), the parametric uncertainty, including that of the overall normalization, α_s and m_t (δ_{param}), and the perturbative scale uncertainty (δ_{scale}):

$$\mathcal{B}(\bar{B} \to X_s\gamma;\, E_\gamma > 1.6\,\text{GeV}) \times 10^4 =$$
$$(3.56\ {}^{+0.24}_{-0.40}\big|_{\frac{m_c}{m_b}} \pm 0.02_{\text{CKM}} \pm 0.24_{\text{param.}} \pm 0.14_{\text{scale}}) \tag{6}$$

$$\mathcal{B}(\bar{B} \to X_d\gamma;\, E_\gamma > 1.6\,\text{GeV}) \times 10^5 =$$
$$(1.36\ {}^{+0.14}_{-0.21}\big|_{\frac{m_c}{m_b}} \pm 0.15_{\text{CKM}} \pm 0.09_{\text{param.}} \pm 0.05_{\text{scale}}) \tag{7}$$

$$\mathcal{B}(\bar{B} \to X_s\gamma;\, E_\gamma > m_b/20) \times 10^4 =$$
$$(3.74\ {}^{+0.26}_{-0.44}\big|_{\frac{m_c}{m_b}} \pm 0.02_{\text{CKM}} \pm 0.25_{\text{param.}} \pm 0.15_{\text{scale}}) \tag{8}$$

$$\mathcal{B}(\bar{B} \to X_d\gamma; E_\gamma > m_b/20) \times 10^5 =$$
$$(1.44\ {}^{+0.15}_{-0.23}\big|_{\frac{m_c}{m_b}} \pm 0.16_{\text{CKM}} \pm 0.10_{\text{param.}} \pm 0.06_{\text{scale}}). \tag{9}$$

The CKM uncertainties are almost negligible in $b \to s\gamma$ transitions but play an important role in $b \to d\gamma$ ones. This implies the large impact on the CKM phenomenology of the latter. We note that in the $b \to d$ mode there is an additional uncertainty due to the *up* quark loops which is suppressed by Λ_{QCD}/m_b (for details see [15]).

The direct CP asymmetries in $\bar{B} \to X_q\gamma$ are defined by

$$A_{CP}^{b\to q\gamma} \equiv \frac{\Gamma[\bar{B} \to X_q\gamma] - \Gamma[B \to X_{\bar{q}}\gamma]}{\Gamma[\bar{B} \to X_q\gamma] + \Gamma[B \to X_{\bar{q}}\gamma]}. \tag{10}$$

It was shown that the CP asymmetry in the $b \to s$ mode is below 1% [20,21,22] within the SM. This small value is a result of three suppression factors. There is an α_s factor needed in order to have a strong phase; moreover, there is a CKM suppression of order λ^2 and there is a GIM suppression of order $(m_c/m_b)^2$, reflecting the fact that in the limit $m_c = m_u$ any CP asymmetry in the SM would vanish. Within the SM the CP asymmetry in the $b \to d$ mode is enhanced, with respect to the one in the $b \to s$ mode, by the CKM factor $[\lambda^2\,((1-\rho)^2 + \eta^2)]^{-1}$.

We update the SM predictions, which are essentially independent of the photon energy cut-off (E_0) and get (for $E_0 = 1.6\,\text{GeV}$):

$$A_{\text{CP}}^{b\to s\gamma} = (0.44\ {}^{+0.15}_{-0.10}\big|_{\frac{m_c}{m_b}} \pm 0.03_{\text{CKM}}{}^{+0.19}_{-0.09}\big|_{\text{scale}})\% \tag{11}$$
$$A_{\text{CP}}^{b\to d\gamma} = (-10.2\ {}^{+2.4}_{-3.7}\big|_{\frac{m_c}{m_b}} \pm 1.0_{\text{CKM}}{}^{+2.1}_{-4.4}\big|_{\text{scale}})\%. \tag{12}$$

The additional parametric uncertainties are subdominant. However, the scale uncertainties are rather large because the CP asymmetries arise at the $O(\alpha_s)$ only. This purely perturbative uncertainty can be removed by a NNLL QCD calculation.

The so-called untagged CP asymmetry $A_{\text{CP}}^{b\to(s+d)\gamma}$ is the favoured observable, at least from the theoretical point of view. A simple expression of this observable is given by

$$A_{\text{CP}}^{b\to(s+d)\gamma} = \frac{A_{\text{CP}}^{b\to s\gamma} + R_{ds}\, A_{\text{CP}}^{b\to d\gamma}}{1 + R_{ds}}\,, \tag{13}$$

$R_{ds} = \Sigma\Gamma_d/\Sigma\Gamma_s,\ \Sigma\Gamma_q := \Gamma(\bar{B} \to X_q\gamma) + \Gamma(B \to X_{\bar{q}}\gamma)$.

As was first noticed in [24], the untagged CP asymmetry vanishes within the SM if the U-spin limit is considered. This is a direct consequence of CKM unitarity. Within the inclusive channels, one can rely on parton–hadron duality and can actually compute the U-spin breaking by keeping a non-vanishing strange quark mass [25]. In [26] U-spin breaking effects were estimated and found to be completely negligible, even beyond the leading partonic contribution within the heavy mass expansion. Thus, the measurement of the untagged CP asymmetry provides a very clean SM test, whether generic new CP phases are active or not. Any significant deviation from the SM zero prediction would be a direct hint of non-CKM contributions to CP violation. An analysis of the untagged asymmetry within various new physics scenarios will be presented in [23].

3 Model-independent formulae

We assume within our model-independent analysis of new physics effects that the dominat ones only modify the Wilson coefficients of the dipole operators $\mathcal{O}_7$ and $\mathcal{O}_8$ and also introduce contributions proportional to the corresponding operators with opposite chirality:

$$\mathcal{O}_7^R = (e/16\pi^2)\, m_b (\bar{q}_R \sigma_{\mu\nu} b_L) F^{\mu\nu}, \tag{14}$$
$$\mathcal{O}_8^R = (g_s/16\pi^2)\, m_b (\bar{q}_R T^a \sigma_{\mu\nu} b_L) G^{a\mu\nu}\ . \tag{15}$$

This is known as a very good approximation for the most relevant new physics scenarios.

Within our model-independent formulae for the branching ratios and CP asymmetries, the Wilson coefficients $C_{7,8(R)}$ and $C_{7,8}$ and all the CKM ratios are left unspecified. The explicit derivation of the formulae given below can be found in [23]. The branching ratio can be written as

$$\mathcal{B}(\bar{B} \to X_q \gamma) = \frac{\mathcal{N}}{100} \left| \frac{V_{tq}^* V_{tb}}{V_{cb}} \right|^2 \mathcal{B}^{\rm unn}\,, \tag{16}$$

where $\mathcal{N} = 2.567\,(1 \pm 0.064) \times 10^{-3}$ is an overall normalization factor, the ratios $R_{7,8}$ and $\widetilde{R}_{7,8}$ are

$$R_{7,8} = \frac{(C_{7,8}^{(0)\rm SM} + C_{7,8}^{(0)\rm NP})(\mu_0)}{C_{7,8}^{(0)\rm SM}(m_t)}, \quad \widetilde{R}_{7,8} = \frac{C_{7,8R}^{(0)\rm NP}(\mu_0)}{C_{7,8}^{(0)\rm SM}(m_t)},$$

and the unnormalized branching ratio is

$$\begin{aligned}\mathcal{B}^{\rm unn} = \Big[& a + a_{77}\,(|R_7|^2 + |\widetilde{R}_7|^2) + a_7^r\,\mathrm{Re}(R_7) + a_7^i\,\mathrm{Im}(R_7) \\ & + a_{88}\,(|R_8|^2 + |\widetilde{R}_8|^2) + a_8^r\,\mathrm{Re}(R_8) + a_8^i\,\mathrm{Im}(R_8) \\ & + a_{\epsilon\epsilon}\,|\epsilon_q|^2 + a_\epsilon^r\,\mathrm{Re}(\epsilon_q) + a_\epsilon^i\,\mathrm{Im}(\epsilon_q) + a_{87}^r\,\mathrm{Re}(R_8 R_7^* + \widetilde{R}_8 \widetilde{R}_7^*) \\ & + a_{7\epsilon}^r\,\mathrm{Re}(R_7 \epsilon_q^*) + a_{8\epsilon}^r\,\mathrm{Re}(R_8 \epsilon_q^*) + a_{87}^i\,\mathrm{Im}(R_8 R_7^* + \widetilde{R}_8 \widetilde{R}_7^*) \\ & + a_{7\epsilon}^i\,\mathrm{Im}(R_7 \epsilon_q^*) + a_{8\epsilon}^i\,\mathrm{Im}(R_8 \epsilon_q^*)\Big]\,. \end{aligned} \tag{17}$$

The CP asymmetry is given by

$$\begin{aligned} A_{\rm CP}^{b\to q\gamma} = & \frac{1}{\mathcal{B}^{\rm unn}}\,\mathrm{Im}\big[a_7^i\,R_7 + a_8^i\,R_8 + a_\epsilon^i\,\epsilon_q \\ & + a_{87}^i\,(R_8 R_7^* + \widetilde{R}_8 \widetilde{R}_7^*) + a_{7\epsilon}^i\,R_7 \epsilon_q^* + a_{8\epsilon}^i\,R_8 \epsilon_q^*\big]\,. \end{aligned} \tag{18}$$

The numerical values of the coefficient functions are collected in Table 1.

Table 1. Numerical values of the coefficients introduced in (17). We give the values corresponding to $E_0 = (1.6\,\mathrm{GeV}, m_b/20)$ and to $m_c/m_b = (0.23, 0.29)$

	NLL			
E_0	1.6 GeV		$m_b/20$	
m_c/m_b	0.23	0.29	0.23	0.29
a	7.8221	6.9120	8.1819	7.1714
a_{77}	0.8161	0.8161	0.8283	0.8283
a_7^r	4.8802	4.5689	4.9228	4.6035
a_7^i	0.3546	0.2167	0.3322	0.2029
a_{88}	0.0197	0.0197	0.0986	0.0986
a_8^r	0.5680	0.5463	0.7810	0.7600
a_8^i	-0.0987	-0.1105	-0.0963	-0.1091
$a_{\epsilon\epsilon}$	0.4384	0.3787	0.8598	0.7097
a_ϵ^r	-1.6981	-2.6679	-1.3329	-2.4935
a_ϵ^i	2.4997	2.8956	2.5274	2.9127
a_{87}^r	0.1923	0.1923	0.2025	0.2025
$a_{7\epsilon}^r$	-0.7827	-1.0940	-0.8092	-1.1285
$a_{8\epsilon}^r$	-0.0601	-0.0819	-0.0573	-0.0783
a_{87}^i	-0.0487	-0.0487	-0.0487	-0.0487
$a_{7\epsilon}^i$	-0.9067	-1.0447	-0.9291	-1.0585
$a_{8\epsilon}^i$	-0.0661	-0.0779	-0.0637	-0.0765

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